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Criterion for Vibrational Freezing in a Nozzle Expansion

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THE flow of a dissociating gas has been studied in some detail, and criteria for the point at which the recombination process freezes have been developed and verified.[1–3] The purpose of this note is to show that the same criterion can be used to predict vibrational freezing as well. This vibrational criterion is checked against numerical data given in Ref. 4.

The criterion of Bray is nearly the same as that of Refs. 1 and 3. For convenience, the later formulation is used as the basis of the present work. The rate equation governing the dissociational relaxation process can be written as

$$d\alpha/dx = -(\alpha - \alpha_e)/r \tag{1}$$

The notation is that of Refs. 2 and 3, where r is a relaxation distance depending on temperature, density, and degree of dissociation α. The subscript e indicates the local equilibrium value. An argument is given to show that the reaction will freeze at the point in the flow where

$$d\alpha_{e\infty}/dx = \alpha_{e\infty}/r_{e\infty} \tag{2}$$

The subscript $e\infty$ indicates the local value for the equilibrium (infinite rate) nozzle flow. In other words, the criterion requires only the equilibrium solution to establish the freezing point.

Define D to be the dissociation energy per mole so that $\alpha D = E_{\text{diss}}$ is the dissociation energy present in the flow. If both sides of Eqs. (1) and (2) are multiplied by D, then they take the form

$$dE_{\text{diss}}/dx = -(E_{\text{diss}} - E_{\text{diss}\,e})/r \tag{3}$$

$$dE_{\text{diss}\,e\infty}/dx = E_{\text{diss}\,e\infty}/r_{e\infty} \tag{4}$$

But Eq. (3) is in the same form that the vibrational relaxation equation usually takes [Eq. (5) of Ref. 4]:

$$dE_{\text{vib}}/dx = -(E_{\text{vib}} - E_{\text{vib}\,e})/r \tag{5}$$

so that it is natural to try the modified form of Eq. (4) to predict the vibrational freezing point:

$$dE_{\text{vib}\,e\infty}/dx = E_{\text{vib}\,e\infty}/r_{e\infty} \tag{6}$$

Reference 4 gives the numerical results for a nozzle flow of a step by step integration of Eq. (5) together with the other fluid dynamical equations to obtain the asymptotic (freeze point) value of the vibrational temperature T_f. For comparison, the freeze point has been determined graphically using Eq. (6), which is used in the form

$$\frac{\tau p_0}{E_{\text{vib}\,e\infty}}\frac{dE_{\text{vib}\,e\infty}}{dt}\frac{r_*}{\tan\theta} = \frac{p_0 r_*}{\tan\theta} \tag{7}$$

where r_* is the throat radius in centimeters, θ is the half angle of the conical nozzle, p_0 is the stagnation pressure in atmospheres, and τ is the vibrational relaxation time. The flow with vibrational equilibrium can be calculated in nondimenional form so that for a given stagnation temperature the stagnation pressure enters only in the parameter $p_0 r_*/\tan\theta$, and the left-hand side of Eq. (7) is a function of temperature only. In plotting the curve, the same gas data and dependence of τ on temperature were used as were used in Ref. 4. The result is shown in Fig. 1. Table 1 gives the comparison between the freeze point temperature as given in Ref. 4 and as determined from Fig. 1. As can be seen, the agreement is quite good considering the fact that both Ref. 4 and the present work have used approximate procedures and fairing to achieve the final result.

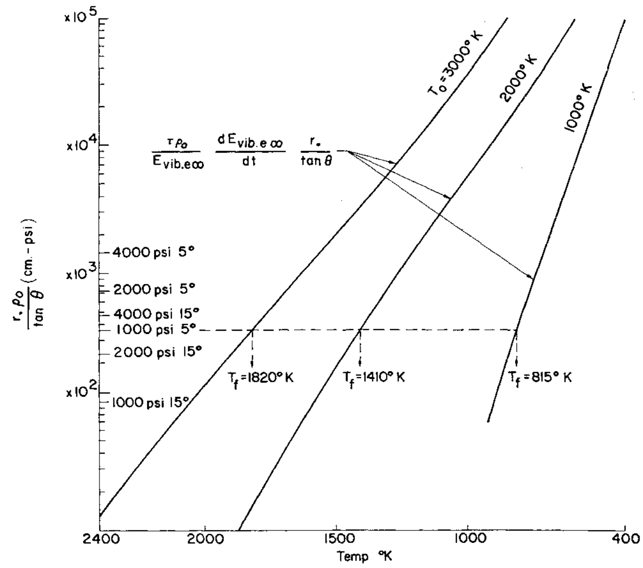

Fig. 1 Curves for the determination of freeze temperature

Table 1 Comparison of exact and approximate values of vibrational freezing temperature

T_0, °K	θ	p_0, psi	T_f (Ref. 4), °K	T_f (present method), °K
3000	5	1000	1908	1820
	5	2000	1752	1690
	5	4000	1612	1560
	15	1000	2140	2205
	15	2000	1995	1900
	15	4000	1853	1775
2000	5	1000	1515	1410
	5	2000	1380	1310
	5	4000	1272	1210
	15	1000	1712	1565
	15	2000	1577	1465
	15	4000	1450	1370
1000	5	1000	912	815
	5	2000	850	760
	5	4000	770	710
	15	1000	955	890
	15	2000	922	845
	15	4000	872	795

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Influence of Calorimeter Heat Transfer Gages on Aerodynamic Heating

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Nomenclature

x	= space variable in the mainstream direction
u	= fluid speed in the x direction
l	= a typical length for the flow
Re	= flow Reynolds number based on this length l and on freestream conditions
$\tau_0(x)$	= wall skin friction
ρ	= local fluid mass density
$q_0(x)$	= rate of conductive heat transfer to the wall
h	= local fluid total enthalpy
g	= $(1 - h/h_e)$, local nondimensional total enthalpy
α	= coefficient in Eq. (2) for skin friction

Subscripts

e	= evaluated external to the boundary layer (in mainstream)
0	= evaluated at the wall
d	= evaluated at a wall-temperature discontinuity
1	= evaluated at the front edge of a gage
2	= evaluated at the rear edge of a gage
x	= based on the length x instead of on l

Analysis

THE center of a calorimeter heat transfer gage such as is described by Rose and Stark[1] reaches, by design, after a short time a temperature lower than that of the surrounding model surface. It is desirable to find the effect of this near-discontinuity in surface temperature on the aerodynamic heating rate measured by the gage.

An analysis first proposed by Lighthill[2] which linearized the boundary layer energy equation is useful here. The linearization is better for small streamwise pressure gradient and for large Prandtl number. Lighthill[2] analyzed only the case of zero streamwise pressure gradient, although Illingworth[3] later analyzed the case of nonzero pressure gradient. Only the constant pressure case is used here to illustrate the influence of a nearly discontinuous wall temperature.

Solution of the linearized energy equation for a given wall-temperature distribution results[2] in the following expression for the heat transfer $q_0(x)$ to the wall:

$$\frac{q_0(x)}{\rho_e(x)u_e(x)h_e(x)} = -\frac{(\frac{2}{3})^{2/3}}{2(\frac{1}{3})!} Re^{-1/3} \left\{\frac{2\tau_0(x)}{\rho_e(x)\ u_e^2(x)}\right\}^{1/2} \times \int_{x_1=0}^{x} dg_0(x_1) \left[\int_{x_1}^{x} \frac{1}{l} \left\{\frac{2\tau_0(x_2)}{\rho_e(x_2)u_e^2(x_2)}\right\}^{1/2} dx_2\right]^{-1/3} \quad (1)$$

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(Note that $(\frac{1}{3})! = 0.8930$.) Lighthill[2] indicates that the accuracy of Eq. (1) may be improved by a suitable alteration of the constant multiplier of its right-hand side.

Application to the Gage Problem

To find the effect of the nearly discontinuous surface-temperature distribution associated with a calorimeter gage, the practical temperature distribution is approximated by one with a discontinuous decrease at the front of the gage and with a discontinuous increase at its rear edge. Such a distribution comprises a continuous one and one that is zero everywhere except on the gage where it has a constant negative value. Since the energy equation had been linearized, the results of applying Eq. (1) to each of these constituent distributions in turn may be added to give the required solution. It also is apparent from Eq. (1) that the contributions from any additional square-wave temperature distributions may be added separately. Such contributions have no upstream influence.

One should realize from the energy equation[2] that the assumed temperature discontinuity at the wall would result in an infinite wall heat transfer rate at the point of discontinuity. This invalidates Lighthill's[2] linearizing assumption at that point. However, the author considers that the solution offered here remains a good approximation for the local heating rate, except at the discontinuity where the nature of the temperature change needs exact specification and an excellent approximation for the averaged gage heating rate.

The wall skin friction for the constant pressure is assumed to be given by

$$2\tau_0(x)/\rho_e(x)u_e^2(x) = \alpha\ Re_x^{-1/2} \quad (2)$$

in which $\alpha \doteq \frac{2}{3}$, as given by Horwarth.[4] At $x = x_d$, a jump in $_0(gx)$ of g_{0d} now is allowed. Then in addition to the wall heating rate arising from the continuous part of the wall-temperature distribution, there is behind the point of discontinuity a contribution of magnitude $\delta q_0(x)$ given by

$$\frac{\delta q_0(x)}{\rho_e(x)u_e(x)h_e(x)} = -\frac{(\frac{3}{8})^{2/3}\ \alpha^{1/3}}{2(\frac{1}{3})!} Re_x^{-1/3} \left\{\frac{2\tau_0(x)}{\rho_e(x)u_e^2(x)}\right\}^{1/2} \times g_{0d} \left\{1 - \left(\frac{x_d}{x}\right)^{3/4}\right\}^{-1/3} \quad (3)$$

The accuracy of Eq. (3) also may be improved by suitable alteration of the constant multiplier of its right-hand side. The infinite heating rate at the assumed temperature discontinuity is apparent from Eq. (3).

Consider now a calorimeter gage mounted between $x = x_1$ and $x = x_2$. The temperature function $g_0(x)$ is taken as one that is constant at $g_0(0^+)$ except on the gage where it is g_{01} lower than elsewhere. In front of the gage the heating rate is unchanged at $q_0(x)$, the value due to $g_0(0^+)$ alone. On the gage the fractional increase in the wall heating rate is

$$\frac{\delta q_0(x)}{q_0(x)} = \frac{g_{01}}{g_0(0^+)} \left\{1 - \left(\frac{x_1}{x}\right)^{3/4}\right\}^{-1/3} \quad (4)$$

and behind the gage it is

$$\frac{\delta q_0(x)}{q_0(x)} = \frac{g_{01}}{g_0(0^+)} \left[\left\{1 - \left(\frac{x_1}{x}\right)^{3/4}\right\}^{-1/3} - \left\{1 - \left(\frac{x_2}{x}\right)^{3/4}\right\}^{-1/3}\right] \quad (4a)$$

In practice, an averaged heating rate of the gage is measured. Eq. (4) is seen to produce a finite average, making the present result physically acceptable. The averaged error obtainable from Eq. (4) is seen to increase directly with the temperature step and to be larger for a gage of small extent in comparison with its distance from the leading edge.